ECON 0150 | Economic Data Analysis

The economist's data analysis skillset.

Part 4.3 | Model Residuals and Diagnostics

General Linear Model

... a flexible approach to run many statistical tests.

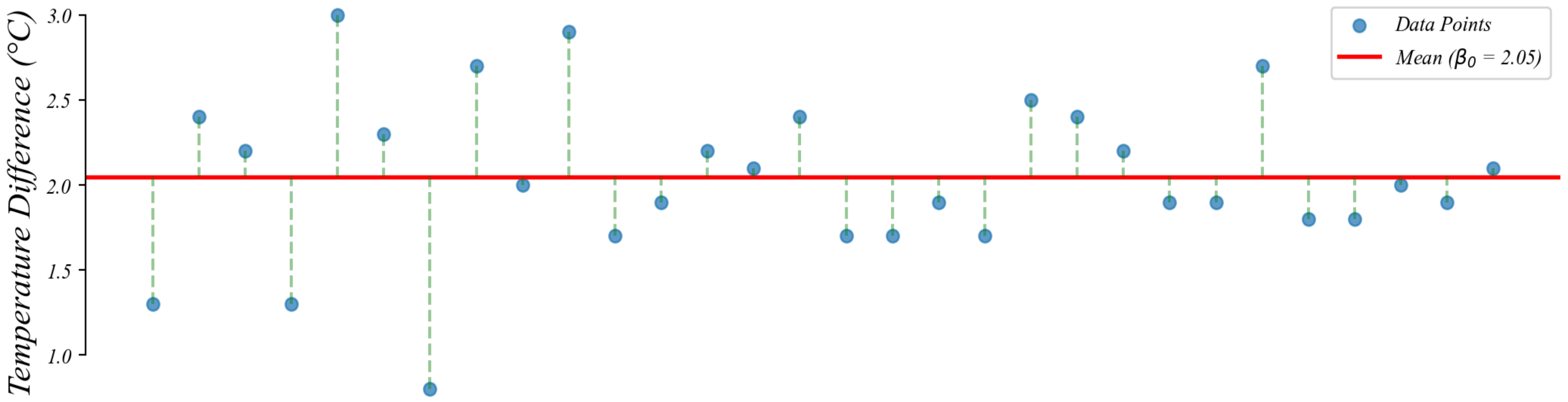
The Linear Model: $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$

- *β_0 is the intercept (value of $\bar{y}$ when $x = 0$)*
- *β_1 is the slope (change in y per unit change in x)*
- *ε_i is the error term (random noise around the model)*

OLS Estimation: Minimizes $\sum_{i=1}^{n} \varepsilon_i^2$

GLM: Intercept Model

A one-sample t-test is a horizontal line model.



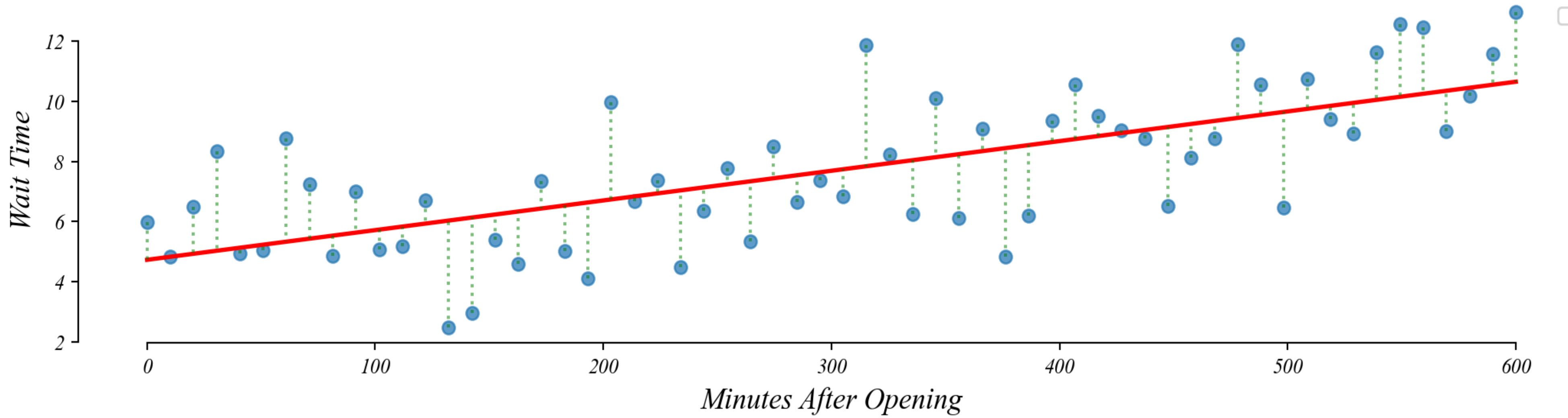
$$Temperature = \beta_0 + \varepsilon$$

> *the intercept β_0 is the estimated mean temperature*

> *the p-value is the probability of seeing β_0 if the null is true*

GLM: Intercept + Slope

A regression is a test of relationships.



$$\text{WaitTime} = \beta_0 + \beta_1 \text{MinutesAfterOpening} + \epsilon$$

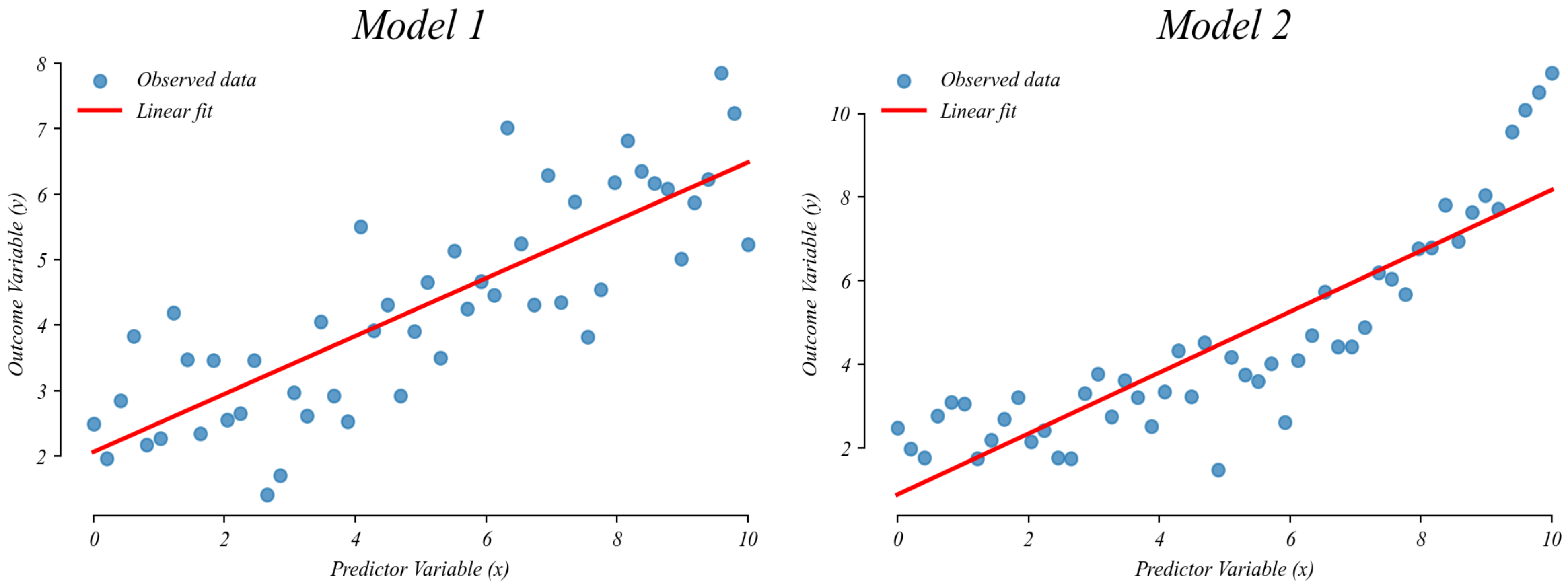
> the intercept parameter β_0 is the estimated temperature at 0 on the horizontal

> the slope parameter β_1 is the estimated change in y for a 1 unit change in x

> the p-value is the probability of seeing parameter (β_0 or β_1) if the null is true

GLM: Intercept + Slope

Which model do you think offers better predictions?



> our model will offer inaccurate predictions if some assumptions aren't met

GLM Assumptions

Our test results are only valid when the model assumptions are valid.

1. ***Linearity***: *The relationship between X and Y is linear*

2. ***Homoskedasticity***: *Equal error variance across all values of X*

3. ***Normality***: *Errors are normally distributed*

4. ***Independence***: *Observations are independent from each other*

GLM Assumptions: why check?

Assumption violations affect our inferences

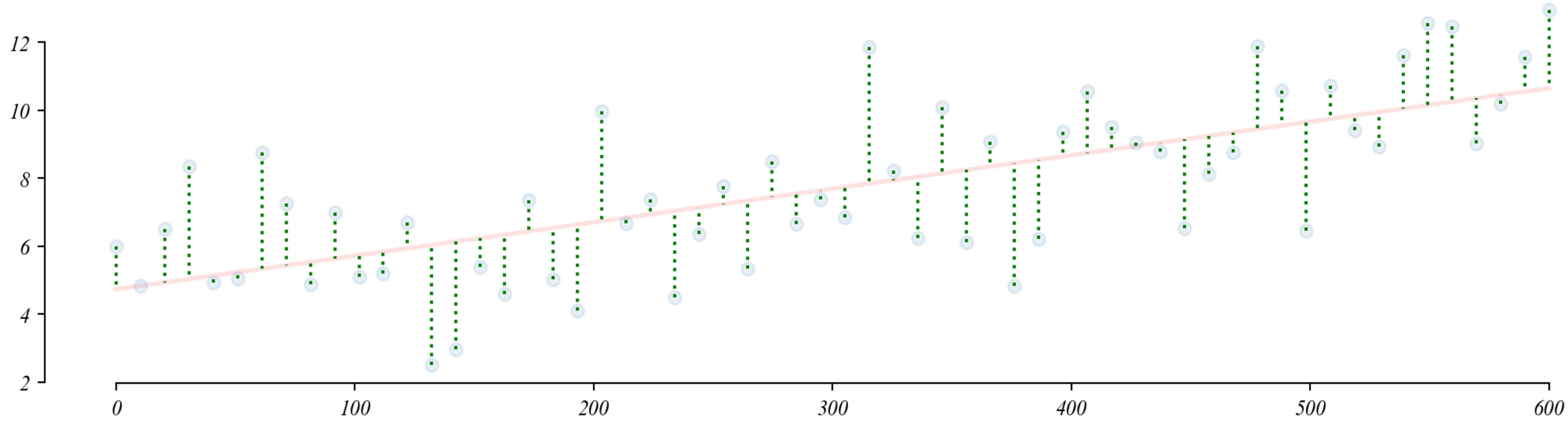
If assumptions are violated:

- *Coefficient estimates may be biased*
- *Standard errors may be wrong*
- *p-values may be misleading*
- *Predictions may be unreliable*

> to test whether the model is 'specified', we can calculate the residuals and the model predictions

Model Residuals

… we can directly examine the error of the model.

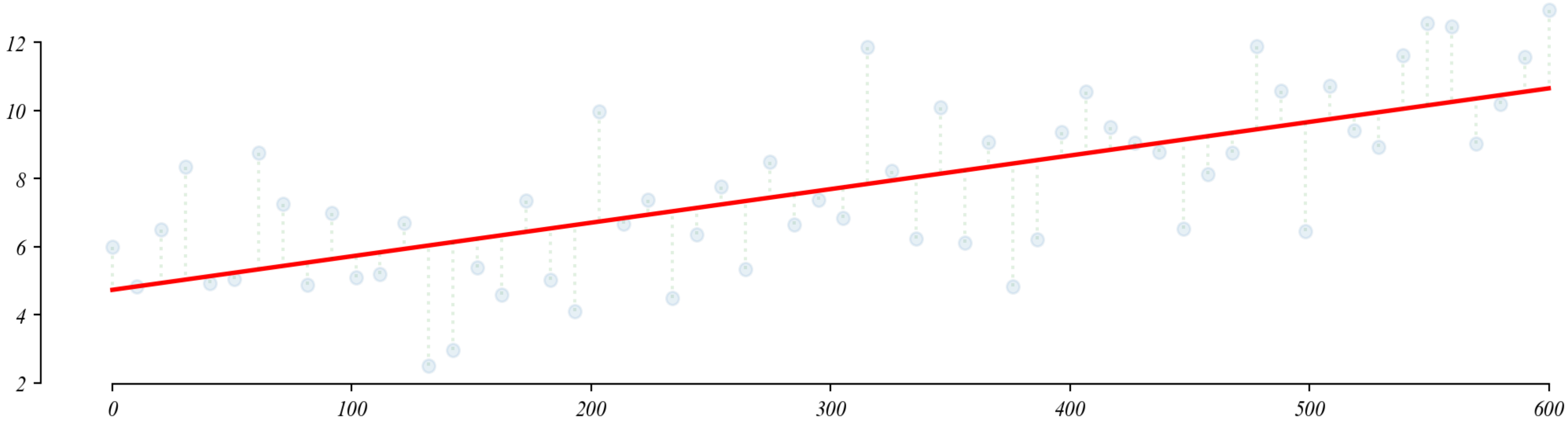



```
# Calculate residuals
residuals = model.resid
sns.histplot(residuals)
```

> *this is ε*

Model Predictions

… we can directly examine the predictions of the model.




```
# Calculate predictions
predictions = model.predict()
sns.histplot(predictions)
```

> *this is $\hat{y}$, the model prediction*

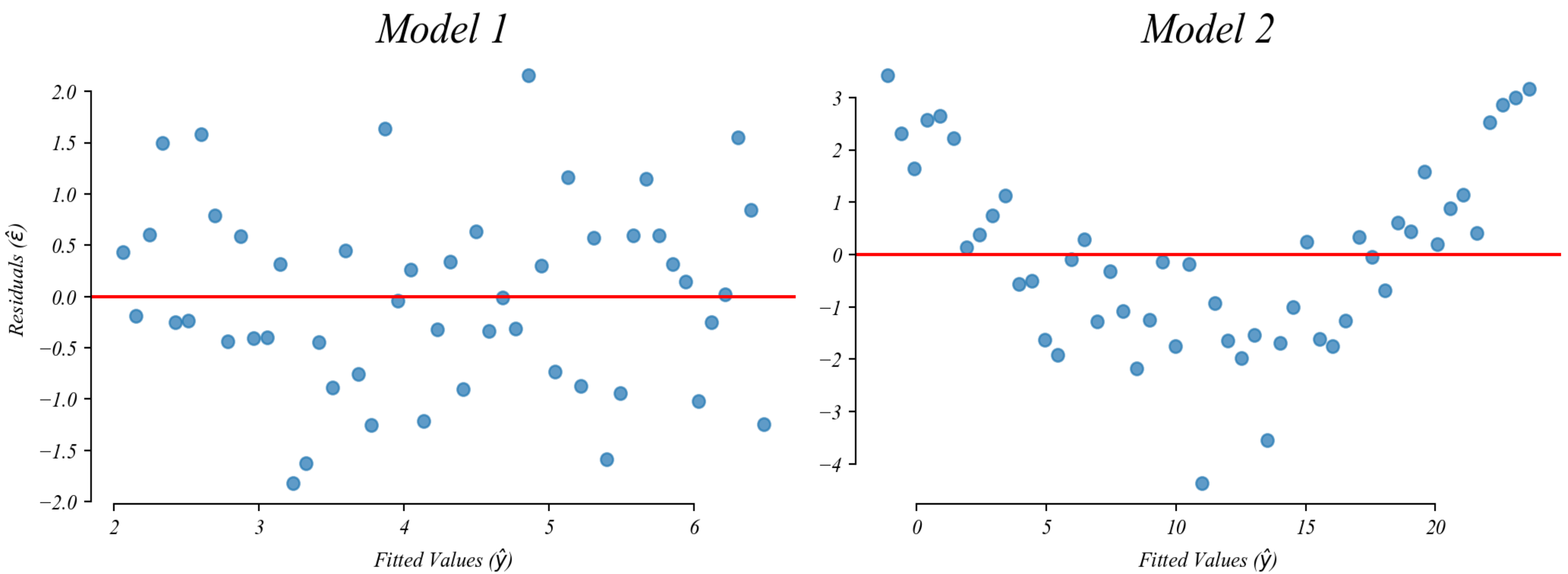
Exercise 4.2 | Residual Plot of Happiness and GDP

*A **Residual Plot** directly visualizes the error for each model estimate.*

```
# Residual Plot: predictions against residuals
plt.scatter(predictions, residuals)
```

Assumption 1: Checking for Linearity

The error term should be unrelated to the fitted value.

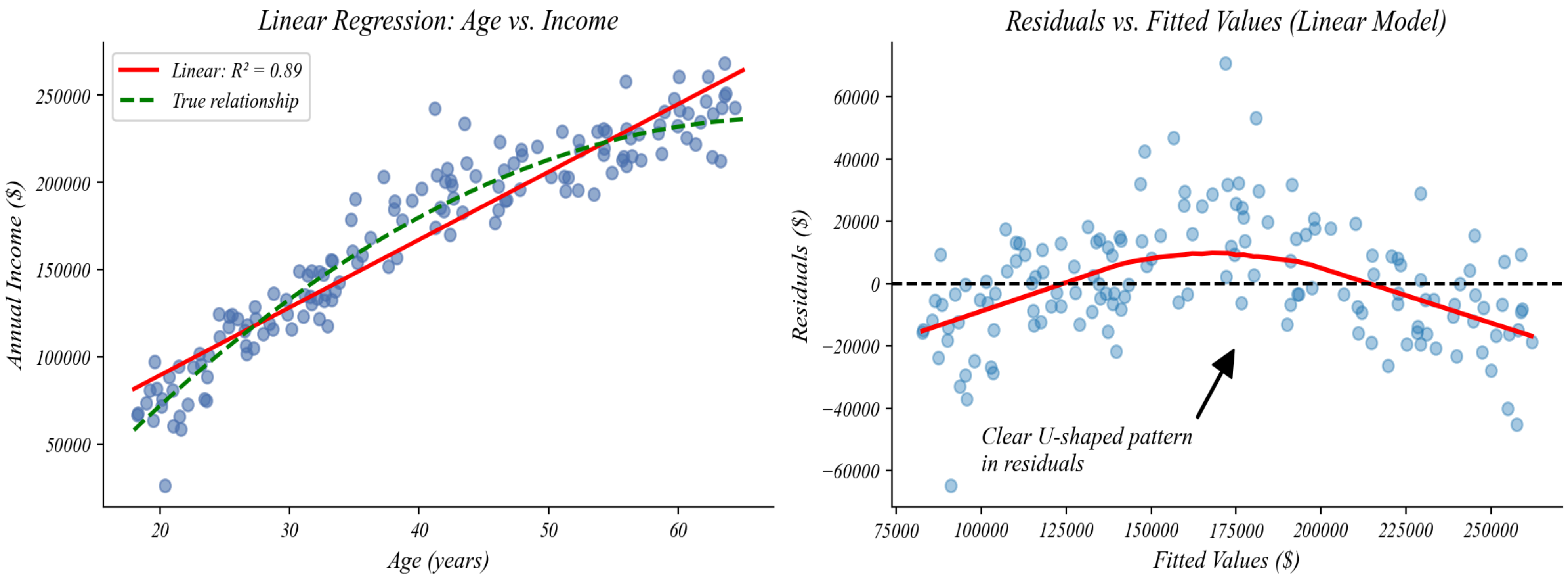


> *the left figure shows that the model is equally wrong everywhere*

> *the right figure shows that the model is a good fit at only some values*

Assumption 1: Checking for Linearity

A non-linear relationship will produce non-linear residuals.



> *linear model misses curvature, leading to systematic errors*

Handling Non-Linear Relationships

Transform variables to become linear

Adding a square term or performing a log transformation can fix the problem.

instead of

$$\text{income} = \beta_0 + \beta_1 \text{age} + \varepsilon$$

we could use

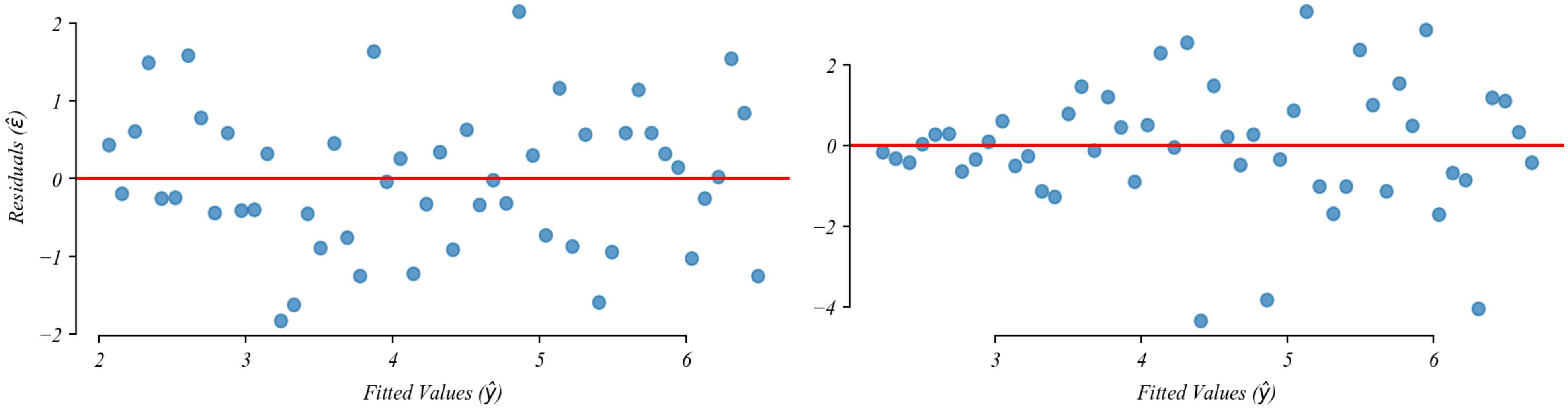
$$\text{income} = \beta_0 + \beta_1 \text{age} + \beta_2 \text{age}^2 + \varepsilon$$

It's also common to log transform either the x or y variable.

Assumption 2: Homoskedasticity

Residuals should be spread out the same everywhere.

Which one of these figures shows homoskedasticity?



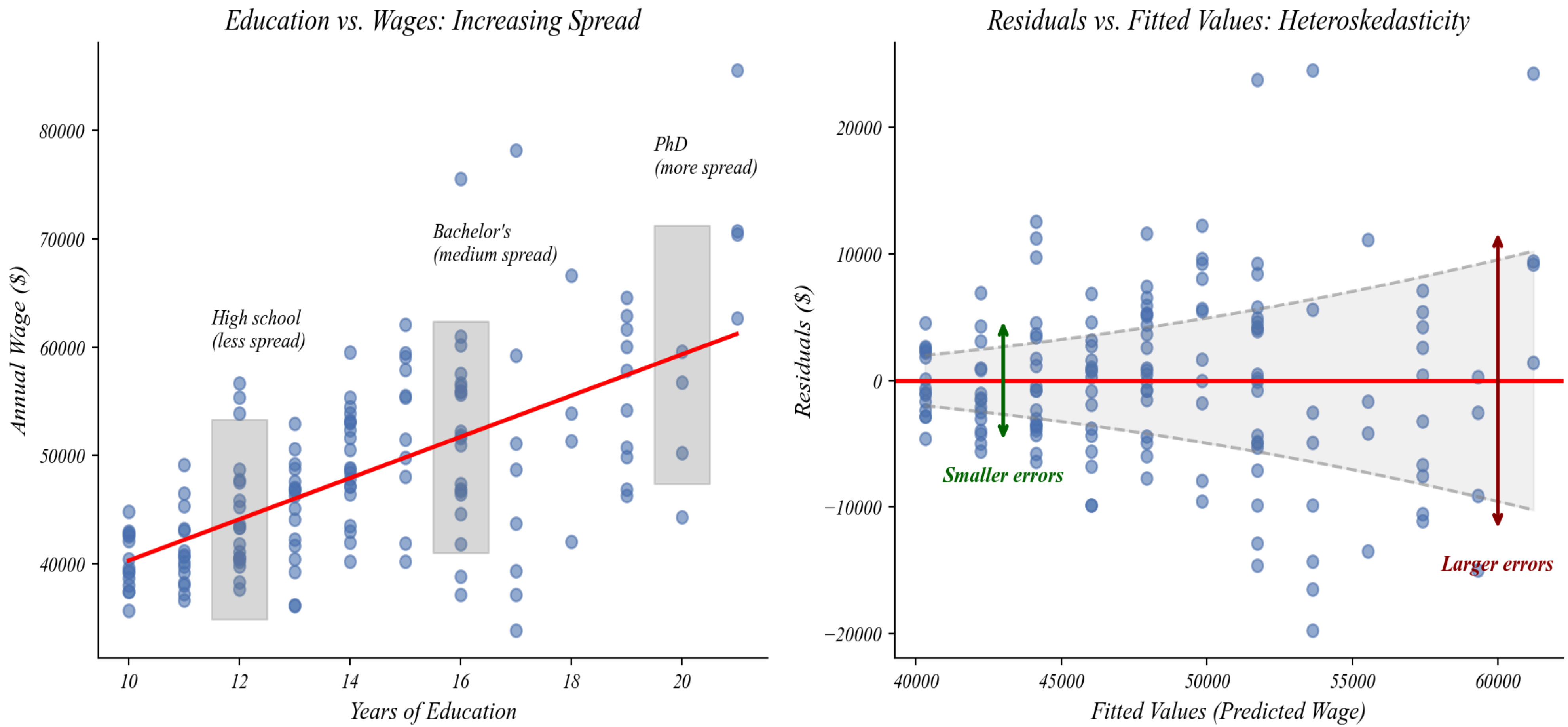
> *the left figure shows constant variability (homoskedasticity)*

> *the right figure shows increasing variability (heteroskedasticity)*

> *residual plots should show that the model is equally wrong everywhere*

Assumption 2: Homoskedasticity

The spread of residuals should not change across values of X.



> *the spread of points increases as education increases*

> *PhD wages vary more than high school wages*

Handling Heteroskedasticity

Robust standard errors give more accurate measures of uncertainty

Robust Standard Errors adjust for the changing spread in our data.

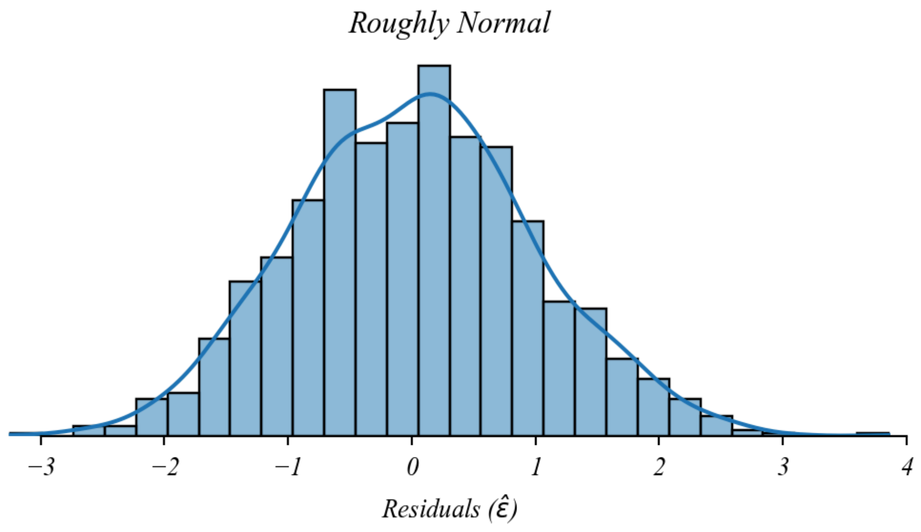
Use robust standard errors to give more accurate hypothesis tests.

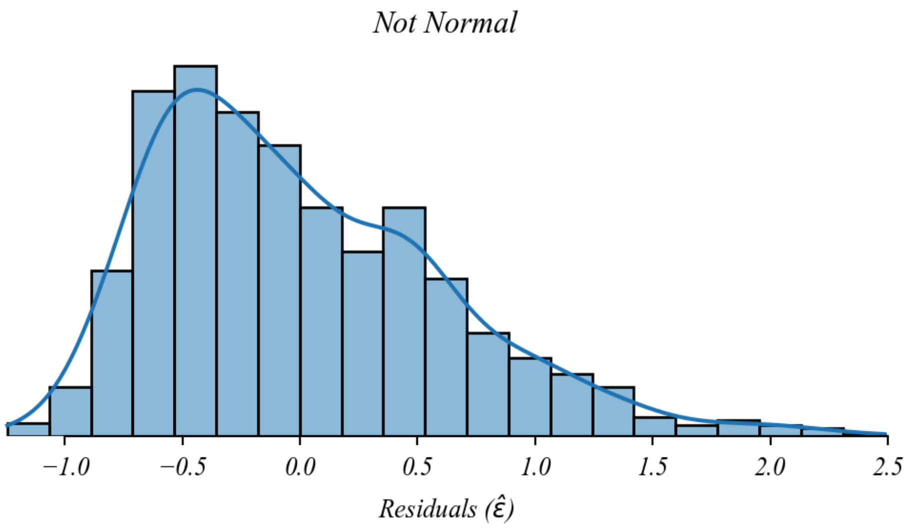
```
# Fit the model with robust standard errors (HC3: heteroskedastic-constant)
robust_model = smf.ols('wages ~ education', data=df).fit(cov_type='HC3')
```

Assumption 3: Normality

Residuals should be normally distributed.

By the CLT we can still use GLM without this *so long as* the sample is large.





Assumption 4: Indepedence

Observations are independent from each other

We’ll return to this assumption in ***Part 4.4 | Timeseries***.

Looking Forward

Extending the GLM framework

Next Up:

- *Part 4.3 | Categorical Predictors*
- *Part 4.4 | Timeseries*
- *Part 4.5 | Causality*

Later:

- *Part 5.1 | Numerical Controls*
- *Part 5.2 | Categorical Controls*
- *Part 5.3 | Interactions*
- *Part 5.4 | Model Selection*

> all built on the same statistical foundation